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> with(Logic):
> with(StringTools):
> with(plots):
> # Anzahl der Pfade
> Anz := (n,k) -> binomial(n-2,k-1)*(k-1)!;
                                 $Anz := (n, k) \mapsto \binom{n-2}{k-1} \cdot (k-1)!$ 

> # n=3
> Anz(3,1) + Anz(3,2);
                                2

> X3 := K12 &or (K23 &and K31);
                                 $X3 := K12 \vee (K23 \wedge K31)$ 

> # KDNF
> X3C := Canonicalize(X3, {K12, K23, K31});
                                 $X3C := (K23 \wedge K31 \wedge K12) \vee (K23 \wedge K31 \wedge (\neg K12)) \vee (K12 \wedge K23 \wedge (\neg K31)) \vee (K12$ 
                                 $\wedge (\neg K23) \wedge K31) \vee (K12 \wedge (\neg K23) \wedge (\neg K31))$ 

> # In String umsetzbare Form erzeugen
> X3B := Export(X3C, form=boolean);
                                 $X3B := K23 \text{ and } K31 \text{ and } K12 \text{ or } K23 \text{ and } K31 \text{ and not } K12 \text{ or } K12 \text{ and } K23 \text{ and not } K31 \text{ or}$ 
                                 $K12 \text{ and not } K23 \text{ and } K31 \text{ or } K12 \text{ and not } K23 \text{ and not } K31$ 

> # In einen String konvertieren
> X3S := convert(X3B, string);
                                 $X3S :=$ 
                                "K23 and K31 and K12 or K23 and K31 and not K12 or K12 and K23 and not K31 or K12
                                and not K23 and K31 or K12 and not K23 and not K31"

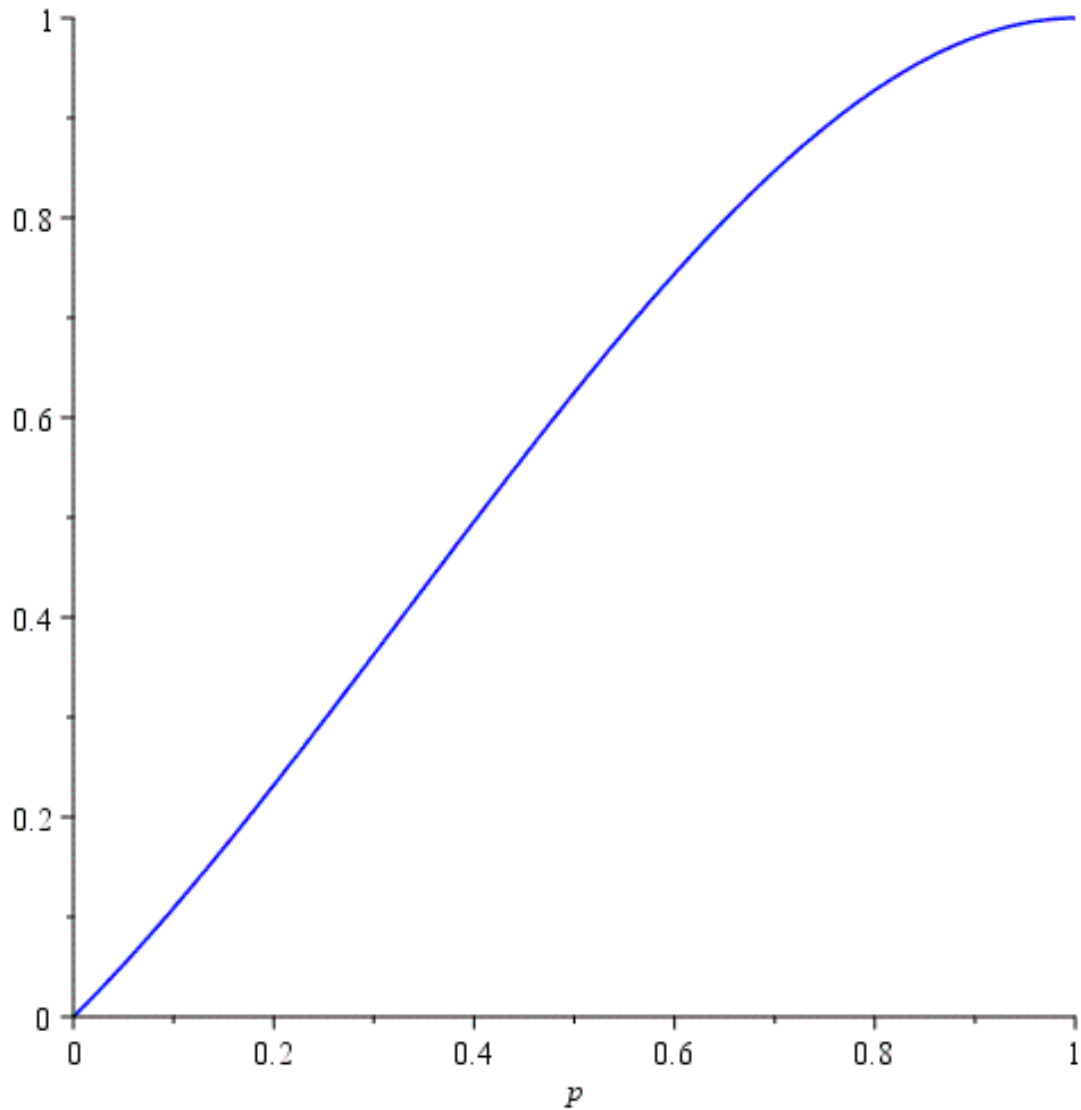
> # Umsetzen in einen String mit arithmetischem Ausdruck
> X3P := Subs({"and" = "*", "or" = "+", "K12" = "p", "K23" = "p", "K31"
= "p", "not K12" = "(1-p)", "not K23" = "(1-p)", "not K31" = "(1-p)"},
X3S);
                                 $X3P := "p * p * p + p * p * (1-p) + p * p * (1-p) + p * (1-p) * p + p * (1-p) * (1-p)"$ 

> # Arithmetischen Ausdruck erzeugen
> X3E := parse(X3P);
                                 $X3E := p^3 + 3p^2(1-p) + p(1-p)^2$ 

> # Arithmetischen Ausdruck vereinfachen
> X3E := simplify(X3E);
                                 $X3E := -p^3 + p^2 + p$ 

> X3G:=plot(X3E, p=0..1, color=blue);

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> # n=4
> Anz(4,1) + Anz(4,2) + Anz(4,3);
5
> X4 := K12 &or (K13 &and K23) &or (K14 &and K24) &or (K14 &and K34
&and K23) &or (K13 &and K34 &and K24);
X4 := (((K12 ∨ (K13 ∧ K23)) ∨ (K14 ∧ K24)) ∨ ((K14 ∧ K34) ∧ K23)) ∨ ((K13 ∧ K34)
∧ K24)
> # KDNF
> X4C := Canonicalize(X4,{K12, K13, K23, K14, K24, K34});

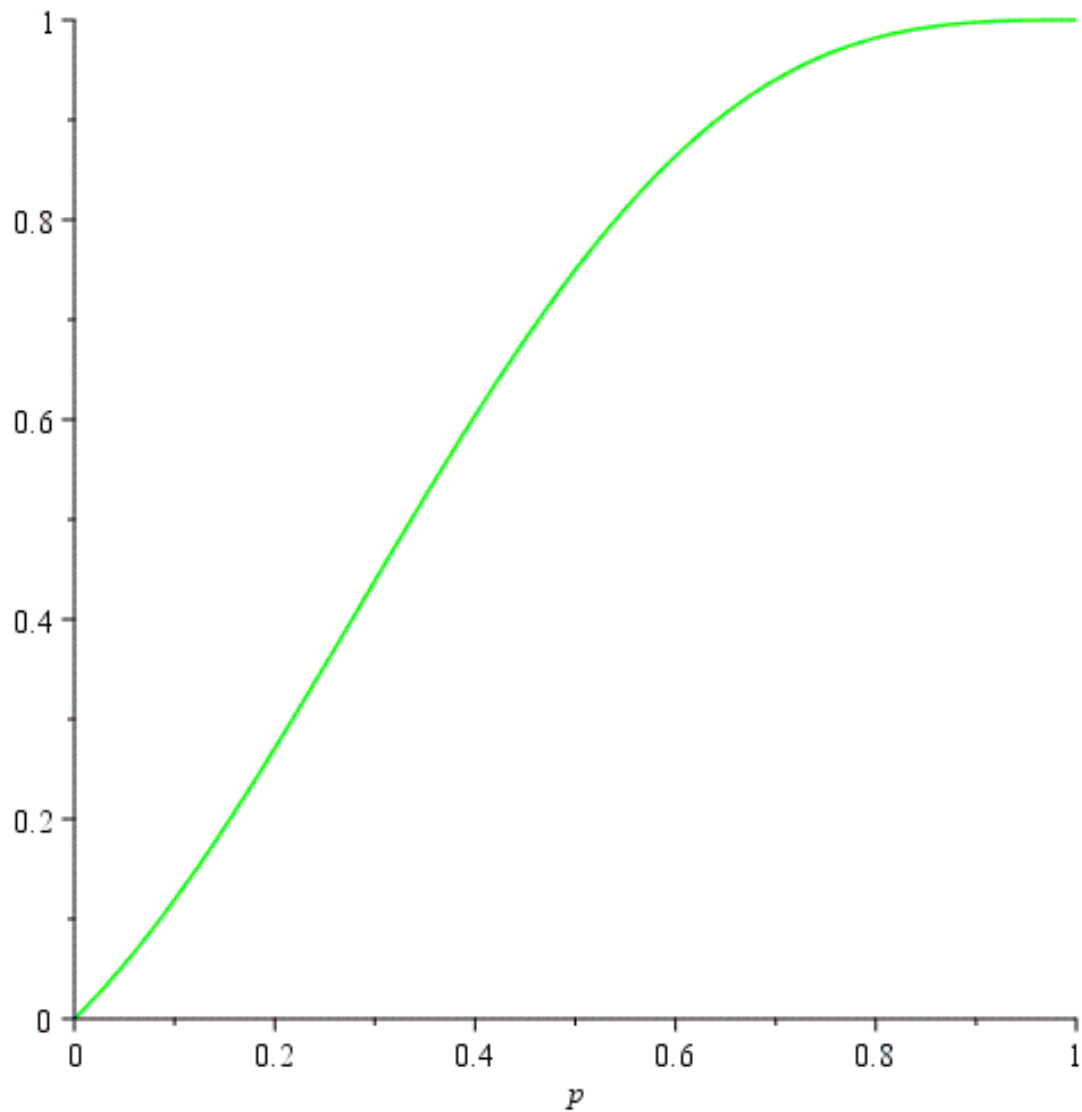
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$$\begin{aligned}
X4C := & (K13 \wedge K23 \wedge K12 \wedge (\neg K14) \wedge (\neg K24) \wedge K34) \vee (K13 \wedge K23 \wedge K12 \wedge (\neg K14) \\
& \wedge (\neg K24) \wedge (\neg K34)) \vee (K13 \wedge K23 \wedge (\neg K12) \wedge K14 \wedge K24 \wedge (\neg K34)) \vee (K13 \\
& \wedge K23 \wedge (\neg K12) \wedge K14 \wedge (\neg K24) \wedge (\neg K34)) \vee (K13 \wedge K23 \wedge (\neg K12) \wedge (\neg K14) \\
& \wedge K24 \wedge (\neg K34)) \vee (K13 \wedge K23 \wedge (\neg K12) \wedge (\neg K14) \wedge (\neg K24) \wedge K34) \vee (K13 \\
& \wedge K23 \wedge (\neg K12) \wedge (\neg K14) \wedge (\neg K24) \wedge (\neg K34)) \vee (K14 \wedge K24 \wedge K12 \wedge K13 \wedge (\\
& \neg K23) \wedge (\neg K34)) \vee (K14 \wedge K24 \wedge K12 \wedge (\neg K13) \wedge K23 \wedge (\neg K34)) \vee (K14 \wedge K24 \\
& \wedge K12 \wedge (\neg K13) \wedge (\neg K23) \wedge K34) \vee (K14 \wedge K24 \wedge K12 \wedge (\neg K13) \wedge (\neg K23) \wedge (\\
& \neg K34)) \vee (K14 \wedge K24 \wedge (\neg K12) \wedge K13 \wedge (\neg K23) \wedge (\neg K34)) \vee (K14 \wedge K24 \wedge (\\
& \neg K12) \wedge (\neg K13) \wedge K23 \wedge (\neg K34)) \vee (K14 \wedge K24 \wedge (\neg K12) \wedge (\neg K13) \wedge (\neg K23) \\
& \wedge K34) \vee (K14 \wedge K24 \wedge (\neg K12) \wedge (\neg K13) \wedge (\neg K23) \wedge (\neg K34)) \vee (K12 \wedge K13 \\
& \wedge K14 \wedge (\neg K23) \wedge (\neg K24) \wedge K34) \vee (K12 \wedge K13 \wedge K14 \wedge (\neg K23) \wedge (\neg K24) \wedge (\\
& \neg K34)) \vee (K12 \wedge K13 \wedge (\neg K14) \wedge (\neg K23) \wedge K24 \wedge (\neg K34)) \vee (K12 \wedge K13 \wedge (\\
& \neg K14) \wedge (\neg K23) \wedge (\neg K24) \wedge K34) \vee (K12 \wedge K13 \wedge (\neg K14) \wedge (\neg K23) \wedge (\neg K24) \\
& \wedge (\neg K34)) \vee (K12 \wedge (\neg K13) \wedge K14 \wedge K23 \wedge (\neg K24) \wedge (\neg K34)) \vee (K12 \wedge (\neg K13) \\
& \wedge K14 \wedge (\neg K23) \wedge (\neg K24) \wedge K34) \vee (K12 \wedge (\neg K13) \wedge K14 \wedge (\neg K23) \wedge (\neg K24) \\
& \wedge (\neg K34)) \vee (K12 \wedge (\neg K13) \wedge (\neg K14) \wedge K23 \wedge K24 \wedge K34) \vee (K12 \wedge (\neg K13) \wedge (\\
& \neg K14) \wedge K23 \wedge K24 \wedge (\neg K34)) \vee (K12 \wedge (\neg K13) \wedge (\neg K14) \wedge K23 \wedge (\neg K24) \\
& \wedge K34) \vee (K12 \wedge (\neg K13) \wedge (\neg K14) \wedge K23 \wedge (\neg K24) \wedge (\neg K34)) \vee (K12 \wedge (\neg K13) \\
& \wedge (\neg K14) \wedge (\neg K23) \wedge K24 \wedge K34) \vee (K12 \wedge (\neg K13) \wedge (\neg K14) \wedge (\neg K23) \wedge K24 \\
& \wedge (\neg K34)) \vee (K12 \wedge (\neg K13) \wedge (\neg K14) \wedge (\neg K23) \wedge (\neg K24) \wedge K34) \vee (K12 \wedge (\\
& \neg K13) \wedge (\neg K14) \wedge (\neg K23) \wedge (\neg K24) \wedge (\neg K34)) \vee (K13 \wedge K24 \wedge K34 \wedge K12 \wedge K14 \\
& \wedge K23) \vee (K13 \wedge K24 \wedge K34 \wedge K12 \wedge K14 \wedge (\neg K23)) \vee (K13 \wedge K24 \wedge K34 \wedge K12 \\
& \wedge (\neg K14) \wedge K23) \vee (K13 \wedge K24 \wedge K34 \wedge K12 \wedge (\neg K14) \wedge (\neg K23)) \vee (K13 \wedge K24 \\
& \wedge K34 \wedge (\neg K12) \wedge K14 \wedge K23) \vee (K13 \wedge K24 \wedge K34 \wedge (\neg K12) \wedge K14 \wedge (\neg K23)) \\
& \vee (K13 \wedge K24 \wedge K34 \wedge (\neg K12) \wedge (\neg K14) \wedge K23) \vee (K13 \wedge K24 \wedge K34 \wedge (\neg K12) \\
& \wedge (\neg K14) \wedge (\neg K23)) \vee (K14 \wedge K23 \wedge K34 \wedge K12 \wedge K13 \wedge (\neg K24)) \vee (K14 \wedge K23 \\
& \wedge K34 \wedge K12 \wedge (\neg K13) \wedge K24) \vee (K14 \wedge K23 \wedge K34 \wedge K12 \wedge (\neg K13) \wedge (\neg K24)) \\
& \vee (K14 \wedge K23 \wedge K34 \wedge (\neg K12) \wedge K13 \wedge (\neg K24)) \vee (K14 \wedge K23 \wedge K34 \wedge (\neg K12) \\
& \wedge (\neg K13) \wedge K24) \vee (K14 \wedge K23 \wedge K34 \wedge (\neg K12) \wedge (\neg K13) \wedge (\neg K24)) \vee (K13 \\
& \wedge K23 \wedge K12 \wedge K14 \wedge K24 \wedge (\neg K34)) \vee (K13 \wedge K23 \wedge K12 \wedge K14 \wedge (\neg K24) \wedge (\\
& \neg K34)) \vee (K13 \wedge K23 \wedge K12 \wedge (\neg K14) \wedge K24 \wedge (\neg K34))
\end{aligned}$$

> X4B := Export(X4C, form=boolean);

*X4B := K13 and K23 and K12 and not K14 and not K24 and K34 or K13 and K23 and K12
and not K14 and not K24 and not K34 or K13 and K23 and not K12 and K14 and K24 and
not K34 or K13 and K23 and not K12 and K14 and not K24 and not K34 or K13 and K23
and not K12 and not K14 and K24 and not K34 or K13 and K23 and not K12 and not K14
and not K24 and K34 or K13 and K23 and not K12 and not K14 and not K24 and not K34
or K14 and K24 and K12 and K13 and not K23 and not K34 or K14 and K24 and K12 and
not K13 and K23 and not K34 or K14 and K24 and K12 and not K13 and not K23 and K34
or K14 and K24 and K12 and not K13 and not K23 and not K34 or K14 and K24 and not
K12 and K13 and not K23 and not K34 or K14 and K24 and not K12 and not K13 and K23
and not K34 or K14 and K24 and not K12 and not K13 and not K23 and K34 or K14 and
K24 and not K12 and not K13 and not K23 and not K34 or K12 and K13 and K14 and not
K23 and not K24 and K34 or K12 and K13 and K14 and not K23 and not K24 and not K34
or K12 and K13 and not K14 and not K23 and K24 and not K34 or K12 and K13 and not
K14 and not K23 and not K24 and K34 or K12 and K13 and not K14 and not K23 and not
K24 and not K34 or K12 and not K13 and K14 and K23 and not K24 and not K34 or K12
and not K13 and K14 and not K23 and not K24 and K34 or K12 and not K13 and K14 and
not K23 and not K24 and not K34 or K12 and not K13 and not K14 and K23 and K24 and
K34 or K12 and not K13 and not K14 and K23 and K24 and not K34 or K12 and not K13
and not K14 and K23 and not K24 and K34 or K12 and not K13 and not K14 and K23 and
not K24 and not K34 or K12 and not K13 and not K14 and not K23 and K24 and K34 or
K12 and not K13 and not K14 and not K23 and K24 and not K34 or K12 and not K13 and
not K14 and not K23 and not K24 and K34 or K12 and not K13 and not K14 and not K23
and not K24 and not K34 or K13 and K24 and K34 and K12 and K14 and K23 or K13 and
K24 and K34 and K12 and K14 and not K23 or K13 and K24 and K34 and K12 and not
K14 and K23 or K13 and K24 and K34 and K12 and not K14 and not K23 or K13 and K24
and K34 and not K12 and K14 and K23 or K13 and K24 and K34 and not K12 and K14
and not K23 or K13 and K24 and K34 and not K12 and not K14 and K23 or K13 and K24
and K34 and not K12 and not K14 and not K23 or K14 and K23 and K34 and K12 and
K13 and not K24 or K14 and K23 and K34 and K12 and not K13 and K24 or K14 and K23
and K34 and K12 and not K13 and not K24 or K14 and K23 and K34 and not K12 and
K13 and not K24 or K14 and K23 and K34 and not K12 and not K13 and K24 or K14 and
K23 and K34 and not K12 and not K13 and not K24 or K13 and K23 and K12 and K14
and K24 and not K34 or K13 and K23 and K12 and K14 and not K24 and not K34 or K13
and K23 and K12 and not K14 and K24 and not K34*

> X4S := convert(X4B, string);



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> # Wendepunkte
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> diff(X4E,p$2);
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$$-60p^4 + 140p^3 - 84p^2 + 4$$

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> f4E := unapply(%,p);
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$$f4E := p \mapsto -60 \cdot p^4 + 140 \cdot p^3 - 84 \cdot p^2 + 4$$

```
> solve(f4E(p)=0,p);
```

$$\begin{aligned}
& 1, \frac{(28900 + 540I\sqrt{6955})^{1/3}}{90} + \frac{142}{9(28900 + 540I\sqrt{6955})^{1/3}} + \frac{4}{9}, \\
& - \frac{(28900 + 540I\sqrt{6955})^{1/3}}{180} - \frac{71}{9(28900 + 540I\sqrt{6955})^{1/3}} + \frac{4}{9} \\
& + \frac{I\sqrt{3} \left(\frac{(28900 + 540I\sqrt{6955})^{1/3}}{90} - \frac{142}{9(28900 + 540I\sqrt{6955})^{1/3}} \right)}{2}, \\
& - \frac{(28900 + 540I\sqrt{6955})^{1/3}}{180} - \frac{71}{9(28900 + 540I\sqrt{6955})^{1/3}} + \frac{4}{9} \\
& - \frac{I\sqrt{3} \left(\frac{(28900 + 540I\sqrt{6955})^{1/3}}{90} - \frac{142}{9(28900 + 540I\sqrt{6955})^{1/3}} \right)}{2}
\end{aligned}$$

> evalf(%);

$$1., 1.235725873 - 2. \times 10^{-10}I, -0.1885382211 - 2.098076212 \times 10^{-10}I, 0.2861456817 \\
+ 3.098076212 \times 10^{-10}I$$

> f4E(0.2861456817);

$$-1. \times 10^{-9}$$

> # n = 5

> Anz(5,1) + Anz(5,2) + Anz(5,3) + Anz(5,4);

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> X5 := K13 &or (K12 &and K23) &or (K14 &and K34) &or (K15 &and K35) &or (K12 &and K25 &and K35) &or (K12 &and K24 &and K34) &or (K15 &and K45 &and K34) &or (K15 &and K25 &and K23) &or (K14 &and K24 &and K23) &or (K14 &and K45 &and K35) &or (K15 &and K25 &and K24 &and K34) &or (K15 &and K45 &and K24 &and K23) &or (K14 &and K45 &and K25 &and K23) &or (K14 &and K24 &and K25 &and K35) &or (K12 &and K24 &and K45 &and K35) &or (K12 &and K25 &and K45 &and K34);

$$\begin{aligned}
X5 := & (((((((((((((K13 \vee (K12 \wedge K23)) \vee (K14 \wedge K34)) \vee (K15 \wedge K35)) \vee ((K12 \\
& \wedge K25) \wedge K35)) \vee ((K12 \wedge K24) \wedge K34)) \vee ((K15 \wedge K45) \wedge K34)) \vee ((K15 \wedge K25) \\
& \wedge K23)) \vee ((K14 \wedge K24) \wedge K23)) \vee ((K14 \wedge K45) \wedge K35)) \vee (((K15 \wedge K25) \wedge K24) \\
& \wedge K34)) \vee (((K15 \wedge K45) \wedge K24) \wedge K23)) \vee (((K14 \wedge K45) \wedge K25) \wedge K23)) \\
& \vee (((K14 \wedge K24) \wedge K25) \wedge K35)) \vee (((K12 \wedge K24) \wedge K45) \wedge K35)) \vee (((K12 \\
& \wedge K25) \wedge K45) \wedge K34)
\end{aligned}$$

> #KDNF

> X5C := Canonicalize(X5, {K13, K12, K23, K14, K34, K15, K35, K25, K24, K45});

> X5B := Export(X5C, form=boolean);


```
> X5S := convert(X5B, string);
```

```
> X5P := Subs({"and" = "*", "or" = "+", "K14" = "p", "K24" = "p", "K12" = "p", "K13" = "p", "K23" = "p", "K34" = "p", "K15" = "p", "K35" = "p", "K25" = "p", "K45" = "p", "not K23" = "(1-p)", "not K34" = "(1-p)", "not K14" = "(1-p)", "not K24" = "(1-p)", "not K13" = "(1-p)", "not K12" = "(1-p)", "not K15" = "(1-p)", "not K35" = "(1-p)", "not K25" = "(1-p)", "not K45" = "(1-p)"}, X5S);  
>
```

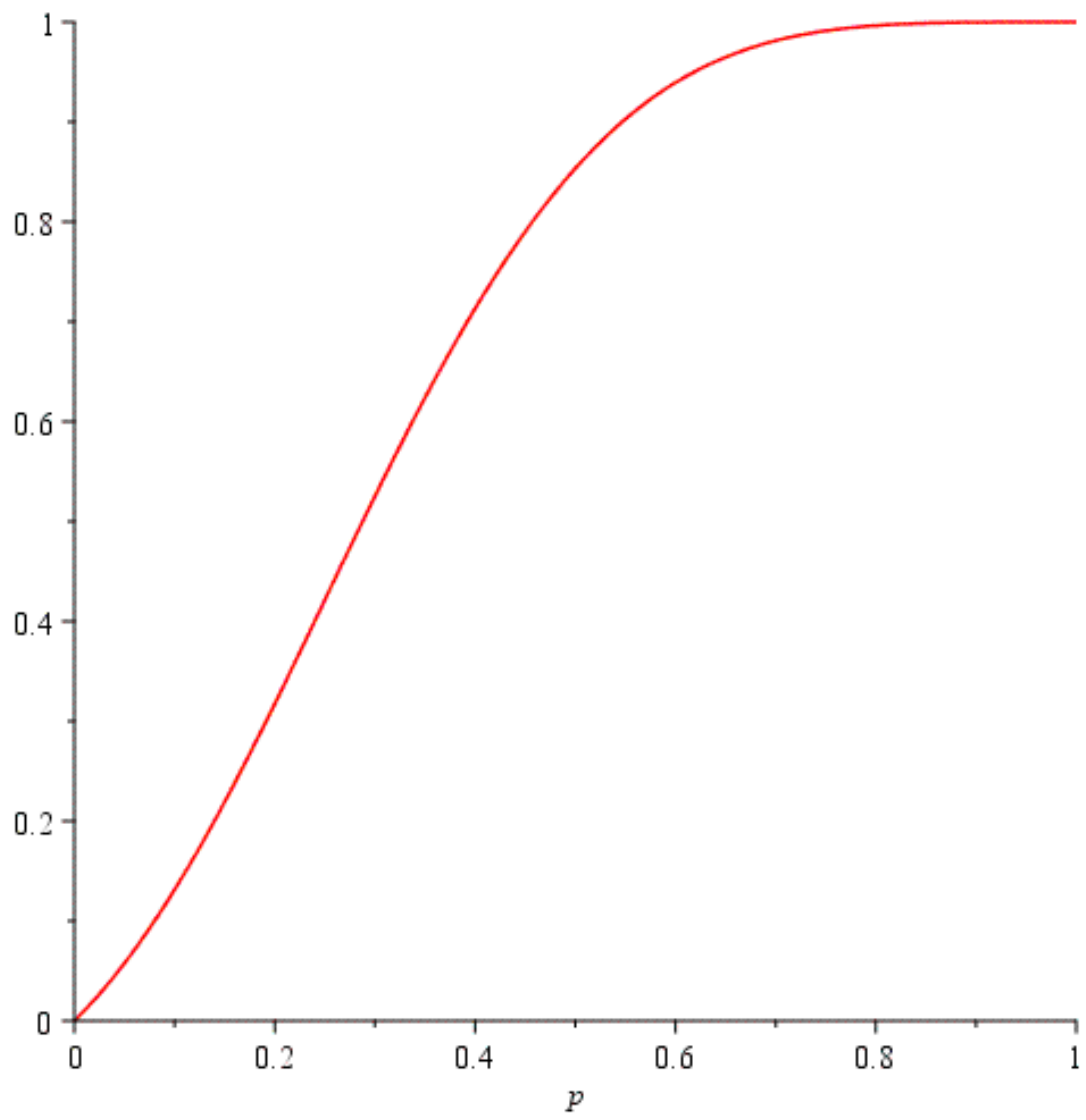
```
> X5E := parse(X5P);
```

$$X5E := p(1-p)^9 + p^{10} + 10p^9(1-p) + 240p^5(1-p)^5 + 63p^3(1-p)^7 + 208p^6(1-p)^4 + 120p^7(1-p)^3 + 45p^8(1-p)^2 + 12p^2(1-p)^8 + 174p^4(1-p)^6$$

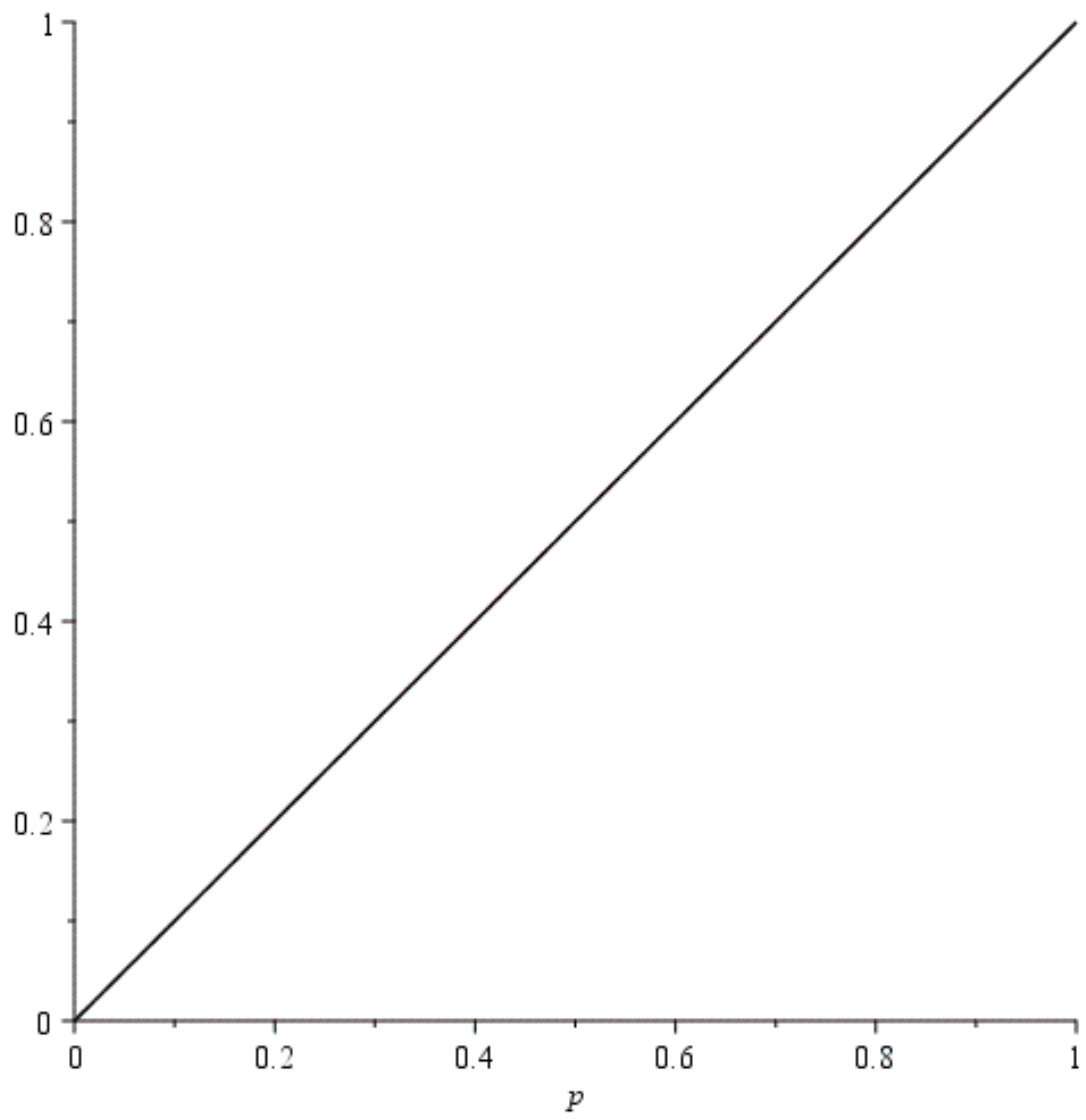
```
> X5E := simplify(X5E);
```

$$X5E := 6p^{10} - 42p^9 + 120p^8 - 175p^7 + 127p^6 - 27p^5 - 15p^4 + 3p^3 + 3p^2 + p$$

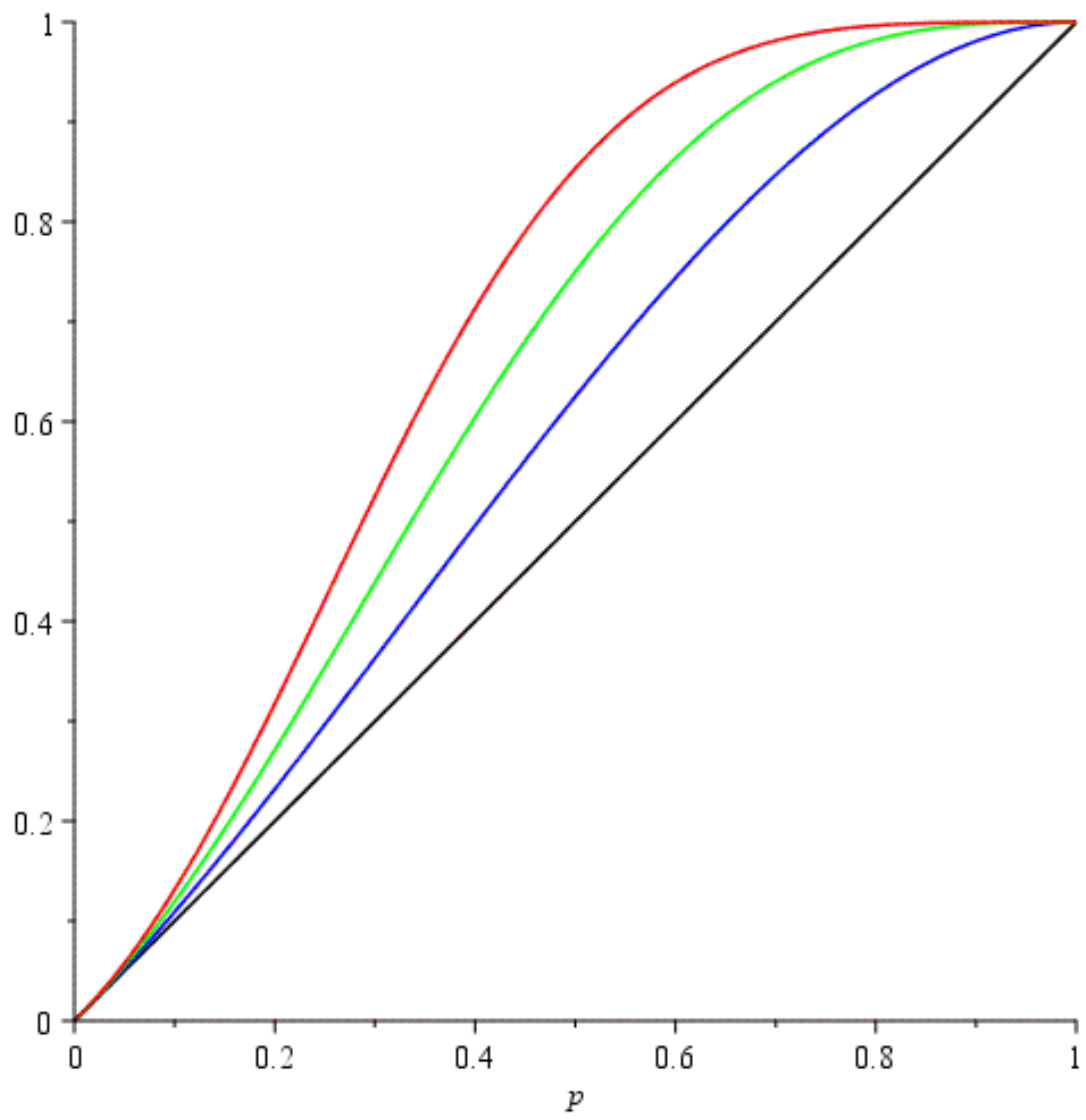
```
> X5G := plot(X5E, p=0..1, color = red);
```



```
> x2G := plot(p,p=0..1,color=black);
```



```
> display(x2G, x3G, x4G, x5G);
```



>